Indian Statistical Institute
II Sem, Mid-Semestral Examination 2008-2009
B.Math.(Hons). III year
Graph Theory and Combinatorics

Date:03-03-2009 Duration: 3 Hours Instructor: N.S.N.Sastry
Max Marks 100

Answer all questions. Your answer should be complete.

1. Let X be the set of all points of the projective space of dimension $n \geq 2$ over $\mathbb{F}_q$ and let $\mathbb{B}$ be the set of all projective subspaces of it of dimension $n-1$.

   (a) Show that $(X, \mathbb{B})$ is a symmetric $2-$ design.

   (b) Show that if the design above is extendable, then $n = 2$.

   $[6 + 10]$

2. Show that a projective plane of order 2 is unique.

   Does there exist a symmetric $t-(v, k, \lambda)$ design for $t \geq 3$? $[6 + 4]$

3. (a) Show that a $3-(41, 4, 1)$ design does not exist, but a $2-(16, 4, 1)$ design exists.Justify.

   (b) If G is a nontrivial group of automorphisms of a $t-(v, k, \lambda)$ design ($t \geq 2$ and $k > \lambda$), show that no nontrivial element of G fixes each block of the design. $[4 + 4 + 7]$

4. Let $(X, \mathbb{B})$ be an extension of a symmetric $2-$design and let $B \in \mathbb{B}$. Let $X_1 = X \backslash B$ and $\mathbb{B}_1 = \{A \in \mathbb{B} : A \cap B = \phi\}$. Show that $(X_1, \mathbb{B}_1)$ is a $2-$ design. Calculate its parameters. $[10]$

5. (a) Obtain the parameters of a symmetric Hadamard design.

   (b) Show that a symmetric Hadamard design is extendable. $[6 + 9]$

6. Let A and B be disjoint subspaces of $\mathbb{F}_q^n$ and let $\dim A = \dim B = k < \frac{n}{2}$. Find the number of maximal subspaces of $\mathbb{F}_q^n$ which contain A but not B. $[10]$

7. (a) Let G be a finite group acting transitively on a finite set X and let H be the stabilizer in G of an element of X. Show that there is a bijection between the orbits of G in $X \times X$ and the orbits of H in X.

   (b) Let V be a vector space of dimension $n + 1 (n \geq 2)$ over a field F and let k be an integer, $1 \leq k \leq n-1$. For the natural action of $GL(V)$ on the set X of projective subspaces of dimension k of the projective space $P(V)$, find the number of orbits of $GL(V)$ on $X \times X$. $[6 + 6]$

8. (a) Find the maximum possible minimum weight of a q-ary linear $[n, k, d]$- code C.

(b) Show that, if a code C as above contains the maximum possible minimum weight, then $C^{\perp}$ also has the maximum possible minimum weight for its parameters. $[6 + 6]$

—————End—————